Research on a Kind of Approximate Method of Solving Fractional Calculus Based on GL Defination

Wenjing Wang[1], Jing Li[2], Hui Li[3] and Junwei Lei[2, a]

[1]The unit 0f A1100000MK00455944, China

[2]College of Weaponry engineering, Naval Engineering University, Wuhan, 430000, China

[3]College of Coastal Defense, Naval Aviation University, Yantai, 264001, China

[a]leijunwei@126.com

Keywords: Fractional order; Simulation; Stability; Control system; Volter equation

Abstract. A kind of approximate method of solving the fractional derivative is proposed and realized with Matlab software and M language to do the numerical simulation. Also the sine function and constant function are taken as an example to testify the proposed method. And we found that if the fractional order is close to 1, then the output is close to the integer order derivative, but if the fractional order is close to zero, then the algorithm need small simulation step to realize the stability of whole system.

Introduction

In engineering, with the continuous development of science and technology and the gradual improvement of target requirements [1-6], practical problems often need to consider the complex situation of the object and various environmental factors, which greatly increases the difficulty of analysis. This urgently requires the establishment of accurate and easy-to-handle mathematical models for complex practical systems [7-14]. The fractional differential equation model provides a new way to solve this problem, because it has a wider application scope than the traditional differential equation model, but only more order parameters of the differential equation.

In recent years, fractional-order systems have attracted much attention in the field of control due to their many practical backgrounds and engineering requirements, and have gradually become a research hot spot [15-18]. It can be roughly divided into two categories: considering fractional order objects and designing effective controllers; and actively introducing fractional order links and utilizing their characteristics to improve control system performance [19-22]. In either case or both, the closed-loop system obtained is a fractional order system. Therefore, the control theory of fractional order systems has become an important and widely concerned new basic subject.

Because of the complexity of fractional order theory research and numerical simulation, this paper is based on the definition of GL to study the numerical simulation of fractional order.

The definition of Fractional Order Differential

Mathematicians have defined fractional calculus from different angles. At present, there are three definitions of fractional calculus including G. L definition, R. L definition and Caputo definition.
The GL definition of fractional differential is defined as

$$\frac{d^q x}{dt^q} = f(t, x) \tag{1}$$

Where q is the order of fractional differential system.

According to the fractional definition, a fractional system can be described as

$$D_t^\alpha f(t) = \lim_{h\to 0} \frac{1}{h^\alpha} \sum_{j=0}^{\frac{t-t_0}{h}} (-1)^j C_j^\alpha f(t-jh) \tag{2}$$

  DOI: 10.25236/isaicn.2019.069

Where

$$C_j^{\alpha} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)} \tag{3}$$

Where the RL definition of fractional order differential is

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)}\left(\frac{d}{dt}\right)^m \int_{\alpha}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau \,, \quad m-1<\alpha<m \tag{4}$$

And the Caputo definition of fractional order differential is

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_{\alpha}^{t} \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau \tag{5}$$

The Approximate Algorithm of Fractional Order Differential

According to the definition of GL fractional order differential, then it can be approximate as following equation

$$D_t^{\alpha} f(t) = \frac{1}{h^{\alpha}} \sum_{j=0}^{\frac{t-t_0}{h}} (-1)^j C_j^{\alpha} f(t-jh) \tag{6}$$

Where

$$C_j^{\alpha} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)} \tag{7}$$

The Simulation Program of Approximate Algorithm

```
clc;close all;clear;
tf=1;h=0.0005;alfa=1/2;
w=waitbar(0,'waitting');
d=1;
for i=0:tf/h
      sf=0;     t=i*h;
      for j=0:i
            Calfaj=gamma(alfa+1)/(gamma(j+1)*gamma(alfa-j+1));
            tjh=t-j*h;
            ftjh=sin(tjh);
            sf=sf+(-1)^j*Calfaj*ftjh;
      end
      dalfaft=1/(h^(alfa))*sf;
      tp(i+1)=t;
      dalfaftp(i+1)=dalfaft;
      d=d+1;
      if d==10
            waitbar(i/(tf/h));
            d=1;
      end
end
close(w)
figure(1)
plot(tp,dalfaftp)
```

Simulation Result and Analysis

The blow figures are the result of fractional order differential with above approximate method for triangular function or constant 2 with fraction order equal to 0.5 or 0.9.

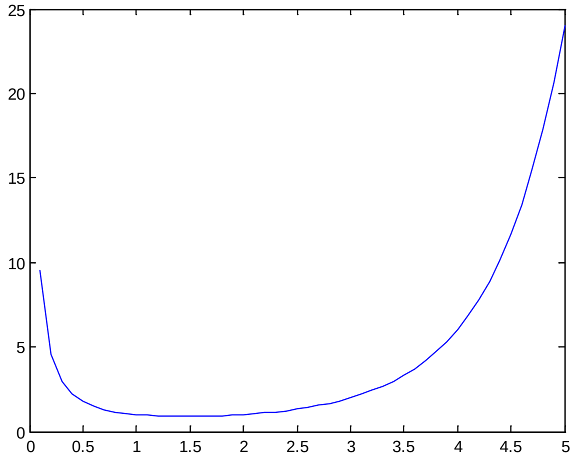

Figure 1 Output curve of order equal to 0.5

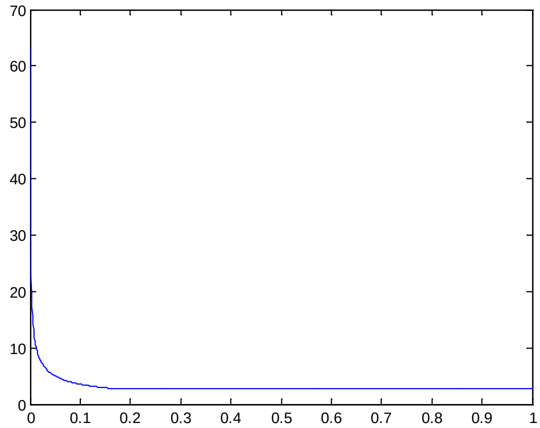

Figure 2 Output curve of constant function

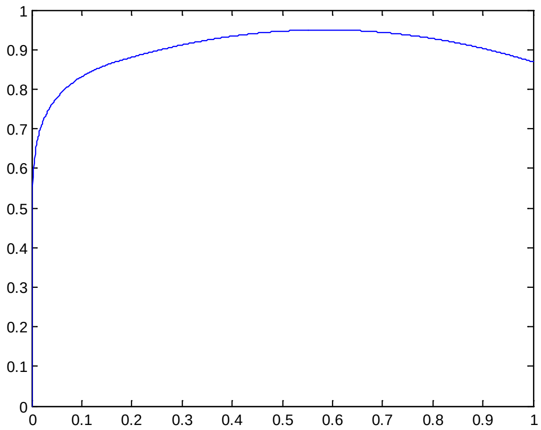

Figure 3 Output curve of order equal to 0.9

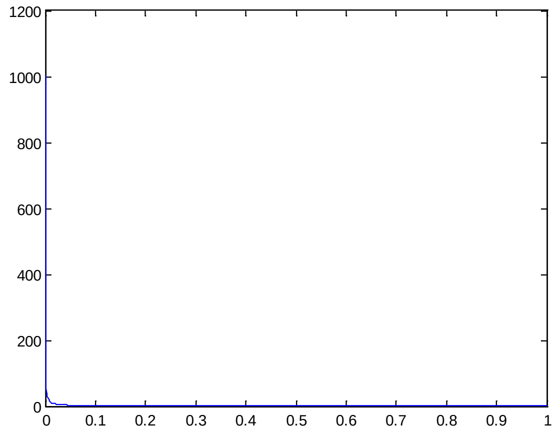

Figure 4 Output curve of constant function and order 0.9

And we found that the fractional order of a constant function is not zero but close to zero. And if the order is close to 1, then the output is close to the derivative of a sine function.

Conclusion

It can be seen that the results of the above calculation are fractional differential, but the calculation of integral is completely different from that of integer integral. Therefore, the fractional differential can only be used for fractional order control, such as fractional order PID control, but it is difficult to be used for fractional order system simulation.

References

[1] L. O. Chua. Memristor-the missing circuit element, IEEE Trans. Circuit Theory 18(5) (1971) 13.

[2] D. B. Strukov, G. S. Snider, D. R. Stewart, R. S. Williams. The missing memristor found, Nature 453 (7191) (2008) 4.

[3] T. He, J.M. Tour. Electronics: The fourth element, Nature 453 (7191) (2008) 2.

[4] M. Di. Ventra, Y.V. Pershin. Experimental demonstration of associative memory with memristive neural networks, Neural Netw. 23 (7) (2010) 6.

[5] G.A. Gibson, S. Musunuru, J. Zhang. An accurate locally active memristor model for s-type negative differential resistance in nbox, Appl. Phys. Lett.

108(2) (2016) 023505.

[6] G.A. Gibson, S. Musunuru, J. Zhang. Nanocsale memristor device as synapse in neuromorphic systems, Nano Lett. 10 (4) (2010) 5.

[7] K. Norris, B.J. Choi, J. Zhang. Trilayer tunnel selectors for memristor memory cells, Adv. Mater. 28 (2) (2016) 7.

[8] H. Kim, S.P. Adhikari, C. Yang. Memristor bridge synapse-based neural network and its learning, Neural Netw. 23 (9) (2012) 10.

[9] Y.M. Banadaki, M.J. Sharifi. General spice models for memristor and application to circuit simulation of memristor-based synapses and memory cells, J. Circuits Syst. Comput. 19 (2) (2010) 18.

[10] I. Grigorenko, E. Grigorenko. Chaotic dynamics of the fractional Lorenz system, Phys. Rev. Lett. 91 (2003) 034101.

[11] J.G. Lu. Chaotic dynamics of the fractional-order Lu system and its synchronization, Phys. Lett. A 354 (2006) 305–311.

[12] Mohammad Rez Faieghi, Hadi Delavari. Chaos in fractional-order Genesio-Tesi system and its synchronization, Commun. Nonlinear Sci. Numer. Simul.17 (2012) 731–741.

[13] W.C. Chen. Nonlinear dynamics and chaos in a fractional-order financial system, Chaos Solitons Fractals 36 (2008) 1305–1314.

[14] Atefeh Marvi Moghadam and Saeed Balochian. Synchronization of economic systems with fractional order dynamics using active sliding mode control, Asian Econ. Financ. Rev. 4 (2014) 692–704.

[15] Z. Wang, X. Huang. Synchronization of a chaotic fractional order economical system with active control, Procedia Eng. 15 (2011) 516–520[14] Khalil, H. K. (2002). Nonlinear systems. New Jersey: Prentice Hall.

[16]Li S. H., Yang J., Chen W. H. & Chen X. (2012). Generalized extended state observer based control for systems with mismatched uncertainties. IEEE Transactions on Industrial Electronics, 59, 4792–4802.

[17]Perruquetti W., Floquet T., & Moulay E. (2008). Finite-time observers: application to secure communication. IEEE Transactions on Automatic Control, 53, 356-360.

[18] Praly L., & Jiang Z. P. (2004). Linear output feedback with dynamic high gain for nonlinear systems. Systems & Control Letters, 53, 107-116.

[19] Rosier L. (1992). Homogeneous Lyapunov function for homogeneous continuous vector field. Systems & Control Letters, 19, 467-473.

[20] Sun B., & Gao Z. (2005). A DSP-based active disturbance rejection control design for a 1-kW H-bridge DC–DC power converter. IEEE Transactions on Industrial Electronics, 52, 1271-1277.

[21] Xia Y. Q., & Fu M. Y. (2013). Compound control methodology for flight vehicles. Berlin: Springer-Verlag.

[22] Levant A. (2003). Higher-order sliding modes, differentiation and output-feedback control. International Journal of Control, 76, 924-941.